

Is Lumpy Investment really Irrelevant for the Business Cycle?*

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Abstract

Smoothness in aggregate capital accumulation is a necessary condition for New-Keynesian (NK) models to imply a quantitatively relevant monetary transmission mechanism (see, e.g., Woodford 2005). Can that aggregate smoothness be entertained in the context of an NK model featuring lumpy plant-level investment? Our answer is yes. Imperfect competition in goods markets and/or sticky prices are identified as economic mechanisms which render lumpy investment relevant in general equilibrium.

Keywords: Lumpy Investment, Sticky Prices.

JEL Classification: E22, E31, E32

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1 Introduction

Does New-Keynesian (NK) theory imply a quantitatively relevant monetary transmission mechanism?¹ Woodford (2005) argues that the answer is yes if aggregate capital accumulation is smooth.² This motivates the following question: Can the required smoothness of aggregate capital accumulation be rationalized under the empirically plausible assumption of lumpy plant-level investment? Our answer is yes. This is surprising given the result obtained by Thomas (2002) in the context of a real business cycle (RBC) model with perfect competition and fully flexible prices. Her analysis implies that the equilibrium dynamics with lumpy plant-level investment are strikingly similar to the ones associated with a specification where investment at the plant level is frictionless.³ In the present paper it is shown that the smoothness of aggregate capital accumulation which is needed for our NK model to imply a quantitatively relevant monetary transmission mechanism can be reconciled with lumpy plant-level investment.

Let us put that result into perspective. Taken at face value Thomas (2002) appears to imply that earlier results stressing the relevance of lumpy investment for aggregate dynamics (see, e.g., Caballero and Engel 1999 and Caballero 1999) hinge somewhat on the use of partial equilibrium frameworks. This is, however, the controversial issue of an ongoing debate. While Khan and Thomas (2004) provide additional robustness analysis in favor of the Thomas (2002) result, Bachmann et al. (2006) and Gourio and Kashyap (2007) argue that lumpy investment matters for aggregate dynamics in the context of RBC models if the latter are calibrated

¹This is a revised version of Norges Bank Working Paper 2005/6, June 16.

²Woodford (2005) obtains smoothness of aggregate capital accumulation from postulating a convex capital adjustment cost at the firm level. Other authors have preferred to engineer aggregate smoothness by assuming a convex investment adjustment cost. See, e.g., Christiano et al. (2005) and Smets and Wouters (2003). See also the discussion in Casares and McCallum (2000), which argues that NK models featuring frictionless endogenous capital accumulation cannot explain the dynamic effects of monetary policy shocks.

³A similar quasi-irrelevance result has been obtained in Veracierto (2002). However, the focus of his analysis is the role of plant-level irreversibility in investment for aggregate fluctuations.

in the ways they advocate. The focus of our analysis is different. It is shown that the presence of monopolistic competition and/or sticky prices in goods markets can alter the irrelevance result in Thomas (2002). Our model is used to disentangle the respective roles of these two economic features for the aggregate relevance of lumpy investment in general equilibrium.

The remainder of the paper is organized as follows. Section 2 lays out a simple theoretical framework for the analysis of the monetary transmission mechanism and, in section 3, our model is used to explain the aggregate consequences of lumpy investment in general equilibrium. Section 4 concludes.

2 Theoretical Framework

Our baseline NK model features lumpy investment. Bernoulli draws are employed both for modeling price stickiness, as is standard in a large body of literature following the lead of Calvo (1983), and for modeling lumpiness in investment, as originally proposed by Kiyotaki and Moore (1997). This way the fact is captured that firms change prices and adjust their capital stocks only infrequently.⁴ Next, a benchmark specification featuring a convex capital adjustment cost at the firm level, as in Woodford (2005), is laid out. Our main result regarding the monetary transmission mechanism holds for any source of aggregate uncertainty and regardless of the particular rule assumed for the conduct of monetary policy. These two aspects of our model are therefore left unspecified.

2.1 Households

Households have access to a complete set of financial securities and supply labor in a perfectly competitive market. A representative household maximizes expected

⁴We are going to refer to the production unit as a firm. There is no distinction between a firm and a plant in the context of our theoretical framework.

discounted utility

$$\sum_{k=0}^{\infty} \beta^k E_t \{U(C_{t+k}, N_{t+k})\}, \quad (1)$$

where E_t denotes an expectation that is conditional on information available through time t . Period utility is denoted $U(\cdot)$, while C_t is a Dixit-Stiglitz composite consumption index, and N_t are hours worked. Throughout, a t subscript is meant to indicate that the corresponding variable is dated as of that time. The period utility function is assumed to be given by

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}, \quad (2)$$

where parameters σ and ϕ are, respectively, the inverse of the household's intertemporal elasticity of substitution and the inverse of the household's labor supply elasticity. The consumption aggregate is defined as follows:

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{for } i \in [0, 1], \quad (3)$$

where parameter $\varepsilon > 1$ measures the elasticity of substitution between the different types of goods, $C_t(i)$.

The household's maximization is subject to a sequence of budget constraints which take the following form

$$\int_0^1 P_t(i) C_t(i) di + E_t \{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t. \quad (4)$$

Here $P_t(i)$ is the price of good i , while $Q_{t,t+1}$ denotes the stochastic discount factor for random nominal payments and D_{t+1} gives the nominal payoff associated with the portfolio held at the end of period t . We have also used the notation W_t for the nominal wage and T_t for dividends resulting from ownership of firms.

Optimizing behavior on the part of households implies the following consumption

demand function for each type of good

$$C_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t, \quad (5)$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index.

The remaining first order conditions read as follows:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t}, \quad (6)$$

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}. \quad (7)$$

The first equation is the labor supply equation, whereas the second is a standard intertemporal optimality condition. Let us finally mention the equilibrium relationship between the gross nominal interest rate, R_t , and the stochastic discount factor:

$$R_t = \frac{1}{E_t\{Q_{t,t+1}\}}.$$

2.2 Firms

There is a continuum of firms indexed on the unit interval.⁵ Each firm $i \in [0, 1]$ is assumed to produce a differentiated good, $Y_t(i)$, using the following Cobb-Douglas production function

$$Y_t(i) = N_t(i)^{1-\alpha} K_t(i)^\alpha, \quad (8)$$

where $\alpha \in [0, 1]$ is the capital share. The variables $N_t(i)$ and $K_t(i)$ denote, respectively, hours used and capital holdings of firm i . The assumption of constant returns to scale is used in order to isolate the respective roles of price stickiness and the market power of firms in explaining the aggregate consequences of lumpy

⁵The fact that entry or exit is not modeled facilitates the calibration since empirical studies of establishment-level investment generally focus on continuing establishments, as Thomas (2002) notes.

investment at the micro level.⁶ The investment good is a Dixit-Stiglitz aggregate of all the goods in the economy with the same constant elasticity of substitution as in the consumption aggregate. Given firm i 's capital stock, $K_t(i)$, the amount of the composite good, $I_t(i)$, that has to be purchased by that firm in order to have a capital stock $K_{t+1}(i)$ in place in the next period is given by

$$I_t(i) = K_{t+1}(i) - (1 - \delta) K_t(i), \quad (9)$$

where parameter δ denotes the depreciation rate. Cost minimization by firms and households implies that demand for each individual good i can be written as follows:

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t^d, \quad (10)$$

where Y_t^d denotes aggregate demand, which is given by

$$Y_t^d = C_t + I_t, \quad (11)$$

and $I_t \equiv \int_0^1 I_t(i) di$ defines aggregate investment.

Each period a measure $(1 - \theta_p)$ of randomly selected firms change their prices and the remaining firms keep their prices constant. Lumpy investment is modeled in an analogous way. In order to capture the fact that firms adjust their capital stocks infrequently, it is assumed that each of them invests in any given period with probability $(1 - \theta_k)$, which is independent of the time elapsed since the last investment. To simplify the analysis two additional assumptions are made. First, the two Bernoulli draws are independent, and, second, the investment lottery is drawn after the price-setting lottery. Hence, firms have to post their prices before

⁶It is worth noting that Thomas (2002) assumes that production units have access to a decreasing returns to scale technology. This economic feature makes her quasi-irrelevance result surprising and interesting.

they get to know the outcome of the investment lottery.

Let us now consider a price setter's problem. Given its capital stock, $K_t(i)$, a price setter i chooses contingent plans for $\{P_{t+j}^*(i), K_{t+j+1}^*(i), N_{t+j}(i)\}_{j=0}^{\infty}$ in order to maximize the following:⁷

$$\sum_{j=0}^{\infty} E_t \{ Q_{t,t+j} [Y_{t+j}^d(i) P_{t+j}(i) - W_{t+j} N_{t+j}(i) - P_{t+j} (K_{t+j+1}(i) - (1 - \delta) K_{t+j}(i))] \}$$

s.t.

$$\begin{aligned} Y_{t+j}^d(i) &= \left(\frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}^d, \\ Y_{t+j}^d(i) &\leq N_{t+j}(i)^{1-\alpha} K_{t+j}(i)^\alpha, \\ I_{t+j}(i) &= K_{t+j+1}(i) - (1 - \delta) K_{t+j}(i), \\ P_{t+j+1}(i) &= \begin{cases} P_{t+j+1}^*(i) & \text{with prob. } (1 - \theta_p), \\ P_{t+j}(i) & \text{with prob. } \theta_p, \end{cases} \\ K_{t+j+1}(i) &= \begin{cases} K_{t+j+1}^*(i) & \text{with prob. } (1 - \theta_k), \\ K_{t+j}(i) & \text{with prob. } \theta_k. \end{cases} \end{aligned}$$

The last restriction reflects our assumption regarding the timing of the two lotteries for price-setting and for investment. Moreover, it is implicit in this formulation that a firm which is not allowed to make an investment decision in a given period is nevertheless assumed to keep its capital constant by paying for the depreciation. This way the fact is captured that firms appear to engage continuously in some small maintenance investment, as Doms and Dunne (1998) report for the U.S. economy. Finally, let us emphasize that, given this structure, a firm's newly set price, $P_{t+j}^*(i)$, will depend on that particular firm's capital stock, $K_{t+j}(i)$, and similarly $K_{t+j+1}^*(i)$ will depend on $P_{t+j}(i)$.

⁷A firm j that cannot change its price at time t solves the same problem, except for the fact that it takes $P_t(j)$ as given.

The first order condition for price-setting is given by

$$\sum_{j=0}^{\infty} \theta_p^j E_t^p \{ Q_{t,t+j} Y_{t+j}^d(i) [P_t^*(i) - \mu P_{t+j} MC_{t+j}(i)] \} = 0, \quad (12)$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ denotes the frictionless mark-up over marginal costs and E_t^p is meant to indicate an expectation that is conditional on the time t state of the world, but integrating only over those future states in which the firm has not reset its price since period t . Finally, $MC_t(i)$ denotes the real marginal cost of firm i in period t . The latter is given by

$$MC_t(i) = \frac{W_t/P_t}{MPL_t(i)}, \quad (13)$$

where $MPL_t(i)$ denotes the marginal product of labor of firm i . Equation (12) reflects that prices are chosen in a forward-looking manner, i.e., taking into account not only current but also future expected marginal costs over the expected lifetime of the chosen price. The only non-standard feature in equation (12) is that capital affects labor productivity and hence a firm's marginal cost.

The first order condition for capital accumulation reads as follows:

$$\sum_{j=0}^{\infty} \theta_k^j E_t^k \{ Q_{t,t+j} [P_{t+j} - Q_{t+j,t+j+1} P_{t+j+1} (MS_{t+j+1}(i) + (1 - \delta))] \} = 0, \quad (14)$$

where E_t^k indicates an expectation that is conditional on the time t state of the world, but integrating only over those future states in which the firm's capital stock is still at the level that was chosen in period t . Finally, $MS_t(i)$ denotes firm i 's real marginal return to capital. It is measured in terms of firm i 's savings in real labor cost associated with having one additional unit of capital in place. The following relationship holds true

$$MS_t(i) = \frac{W_t}{P_t} \frac{MPK_t(i)}{MPL_t(i)}, \quad (15)$$

where $MPK_t(i)$ denotes the marginal product of capital of firm i . The intuition

behind equation (14) is simple and analogous to the one behind the first order condition for price-setting. Firms invest in a forward-looking manner, i.e., by taking into account their future marginal returns to capital over the expected lifetime of the chosen capital stock.

2.3 Market Clearing

Clearing of the labor market requires that hours worked, N_t , are given by the following equation, which holds for all t

$$N_t = \int_0^1 N_t(i) di. \quad (16)$$

Finally, market clearing for each variety i requires at each point in time that

$$Y_t(i) = Y_t^d(i). \quad (17)$$

2.4 Linearized Equilibrium Conditions

A linear approximation to the equilibrium dynamics around a zero inflation steady state is derived. Throughout, lowercase letters denote log-deviations of the original variables from their steady-state values, except for the nominal interest rate, $i_t \equiv \log R_t$, and inflation, $\pi_t \equiv \log \left(\frac{P_t}{P_{t-1}} \right)$.

2.4.1 Households

The household's problem implies a consumption Euler equation and a labor supply equation. They read as follows:

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho), \quad (18)$$

$$\omega_t = \phi n_t + \sigma c_t, \quad (19)$$

where parameter $\rho \equiv -\log \beta$ is the time discount rate, and $\Omega_t \equiv \frac{W_t}{P_t}$ is the real wage.

2.4.2 Firms

The method developed in Woodford (2005) is used to derive the law of motion of aggregate capital and the inflation equation implied by our model. They are given by

$$\Delta k_{t+1} = \beta E_t \{\Delta k_{t+2}\} + \frac{1}{\eta_l} E_t \{\beta (\rho + \delta) m s_{t+1} - (i_t - \pi_{t+1} - \rho)\}, \quad (20)$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa_l m c_t, \quad (21)$$

where Δ is the first-difference operator and η_l and κ_l are parameters which are computed numerically. Moreover, $MS_t \equiv \int_0^1 MS_t(i) di$ denotes the average real marginal return to capital and $MC_t \equiv \int_0^1 MC_t(i) di$ is the average real marginal cost.⁸

Aggregating and log-linearizing the production functions of individual firms (8) results in

$$y_t = (1 - \alpha) n_t + \alpha k_t, \quad (22)$$

where $Y_t \equiv N_t^{1-\alpha} K_t^\alpha$ is aggregate production, up to the first order.

2.4.3 Market clearing

Aggregating and log-linearizing the goods market clearing condition for each variety (17), and invoking (8), (10), and (11), one obtains

$$y_t = \zeta c_t + \frac{1 - \zeta}{\delta} [k_{t+1} - (1 - \delta) k_t], \quad (23)$$

⁸For a derivation of the last two equations in the text, see Appendix A.

where $\zeta \equiv 1 - \frac{\delta\alpha}{\mu(\rho+\delta)}$ denotes the steady-state consumption-to-output ratio, and $\frac{(1-\zeta)}{\delta}$ is the steady-state capital-to-output ratio.

2.5 The Convex Capital Adjustment Cost Case

In what follows a benchmark model featuring a convex capital adjustment cost at the firm level, as in Woodford (2005), is considered.⁹ He assumes the following restriction on capital accumulation

$$I_t(i) = I\left(\frac{K_{t+1}(i)}{K_t(i)}\right) K_t(i), \quad (24)$$

where $I_t(i)$ denotes the amount of the composite good which needs to be purchased by firm i in order to change its capital stock from $K_t(i)$ to $K_{t+1}(i)$ in the next period.¹⁰ Moreover, function $I(\cdot)$ is assumed to be increasing and convex. It is also assumed that this function satisfies the following: $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \eta_c$. Parameter $\eta_c > 0$ measures the convex capital adjustment cost in a log-linear approximation to the equilibrium dynamics.

The linearized equilibrium conditions implied by the benchmark model are identical to the ones associated with the lumpy investment model, except for the inflation equation and the law of motion of capital. The latter two equations read

$$\Delta k_{t+1} = \beta E_t \{\Delta k_{t+2}\} + \frac{1}{\eta_c} E_t \{\beta(\rho + \delta) m s_{t+1} - (i_t - \pi_{t+1} - \rho)\}, \quad (25)$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa_c m c_t, \quad (26)$$

where κ_c is computed numerically.¹¹

⁹The desirability of modeling firm-specific capital accumulation for the analysis of the monetary transmission mechanism is also emphasized by Woodford (2003, Ch. 5).

¹⁰As in our baseline model, it is assumed that the investment good is a Dixit-Stiglitz aggregate of all of the goods in the economy with the same constant elasticity of substitution as in the consumption aggregate.

¹¹Deriving the last two equations is a straightforward application of Woodford's (2005) method.

A comparison of the last two equations with their counterparts (20) and (21) in the lumpy investment model reveals that a model featuring a convex capital adjustment cost at the firm level is observationally equivalent (up to the first order) to our specification with lumpy investment: For any given value of the convex adjustment cost parameter, η_c , there exists a value of the lumpiness parameter, θ_k , such that the two laws of motion of capital implied by the two models are identical. Moreover, the two associated inflation equations coincide for this choice of θ_k .¹² This makes it possible to compare our lumpy investment baseline model with the convex capital adjustment cost benchmark case in a particularly clean way. One possible interpretation of this finding is that it generalizes the well known equivalence result in Rotemberg (1987) to a setting in which two decisions are made simultaneously at the firm level. Interestingly, our analysis reveals that the particular value of θ_k for which the above mentioned equivalence obtains depends on the price stickiness parameter, θ , and on the elasticity of substitution between goods, ϵ , as will be discussed below.

3 Numerical Results

The period length is one quarter. Table 1 shows the baseline parameter values for the lumpy investment model.

Table 1: Baseline Parameter Values

σ	ε	α	β	ϕ	θ_p	θ_k
2	11	0.36	0.99	1	0.75	0.92

For details see Sveen and Weinke (2005).

¹²For a formal proof see Appendix B.

The values assigned to parameters σ , ε , α , β , ϕ , and θ_p are standard.¹³ The baseline value of the lumpiness parameter, θ_k , is 0.92. This appears to be in line with the micro evidence on plant-level investment reported by Doms and Dunne (1998). They use U.S. data on 13,700 manufacturing plants over the 17 year period 1972 to 1988. For each plant they establish a rank distribution of capital growth rates and compute the associated mean and median over all firms for each rank. They find that “many plants experience a few periods of intense capital growth and many periods of relatively small capital adjustment: of the 16 capital growth rate ranks, 12 possess means or medians between -10 and +10%.” Moreover they report that plants choose to change their capital holdings by at least 5% on average every second year. The empirically plausible range for the lumpiness parameter, θ_k , is therefore taken to be the interval (0.88, 0.94). Values in that range imply that production units invest on average about every 2 to 4 years. Our baseline value for the lumpiness parameter simply corresponds to the midpoint of that range, i.e., it implies about 3 years for the average expected time between investments at the plant level.¹⁴

These preparations allow us to address the main question which is asked in the present paper: Can lumpy investment at the micro level be reconciled, under empirically plausible assumptions, with the degree of smoothness in aggregate capital accumulation which is needed to render NK models capable of explaining the dynamic effects of monetary policy shocks? Our answer is yes. A value of about 3 for parameter η_l is needed in order to account for the smooth response of aggregate demand in response to monetary policy shocks, as Woodford (2005) argues in the context of a model featuring a convex capital adjustment cost at the firm level. Given the equivalence between the convex adjustment cost model and our speci-

¹³See, e.g., Sveen and Weinke (2005) and the references therein.

¹⁴This means that the “relatively small capital adjustment” is interpreted as variation in maintenance. Variation in maintenance could be entertained in our theoretical model by allowing the rate of depreciation to be stochastic.

cation with lumpy investment it can be asked what is the corresponding value of the lumpiness parameter needed to entertain this level of aggregate smoothness of capital accumulation and whether or not this value falls within the interval that is considered to be empirically plausible. The result is shown in Figure 1: Woodford’s preferred calibration of the smoothness in aggregate capital accumulation falls well within the empirically plausible range. Specifically, $\eta_l = 3$ is associated with $\theta_k = 0.924$ if the remaining parameters are held constant at their baseline values.

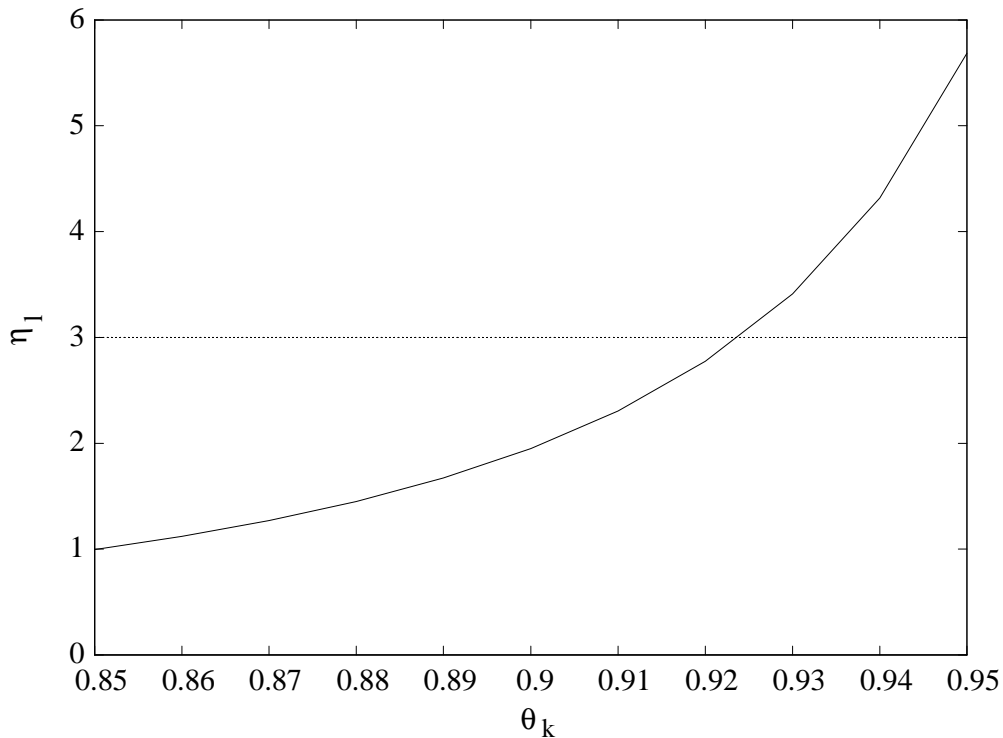


Figure 1: Firm-level lumpiness and aggregate smoothness of capital accumulation.

The result illustrated in Figure 1 is in stark contrast with the findings in Thomas (2002). In the context of her model the implied equilibrium dynamics with lumpy investment are strikingly similar to the ones associated with a specification where investment at the level of the production unit is frictionless. What is the economic reason for this difference in the predictions of RBC and NK theory? The answer is

that price stickiness and the market power of firms, two features that are absent in Thomas's RBC model, affect the smoothness of aggregate capital accumulation with lumpy investment. Our intuition is as follows. With lumpy investment the dynamics of aggregate capital accumulation are driven by the decisions of only a *fraction* of firms. These firms internalize the consequences of their investment decisions for their future expected real returns to capital. In particular, an investing firm foresees that an increase in its capital stock is associated with a decrease in its expected future real return. This means that in response to an increase in the *average* real return to capital, an investing firm will choose to limit the size of its investment if the associated decrease in its own real return is large.¹⁵ The extent to which an investing firm's real marginal return to capital decreases if the capital stock is increased depends in turn on price-setting behavior. The latter is affected by price stickiness and the market power of firms.

First, the role of price stickiness is analyzed under the assumption that the remaining parameters are held constant at their baseline values. The results are shown in Figure 2.

¹⁵The intuition is similar to the one developed by Sbordone (2002) and Galí et al. (2001) for the difference in price-setting behavior under constant and decreasing returns to scale.

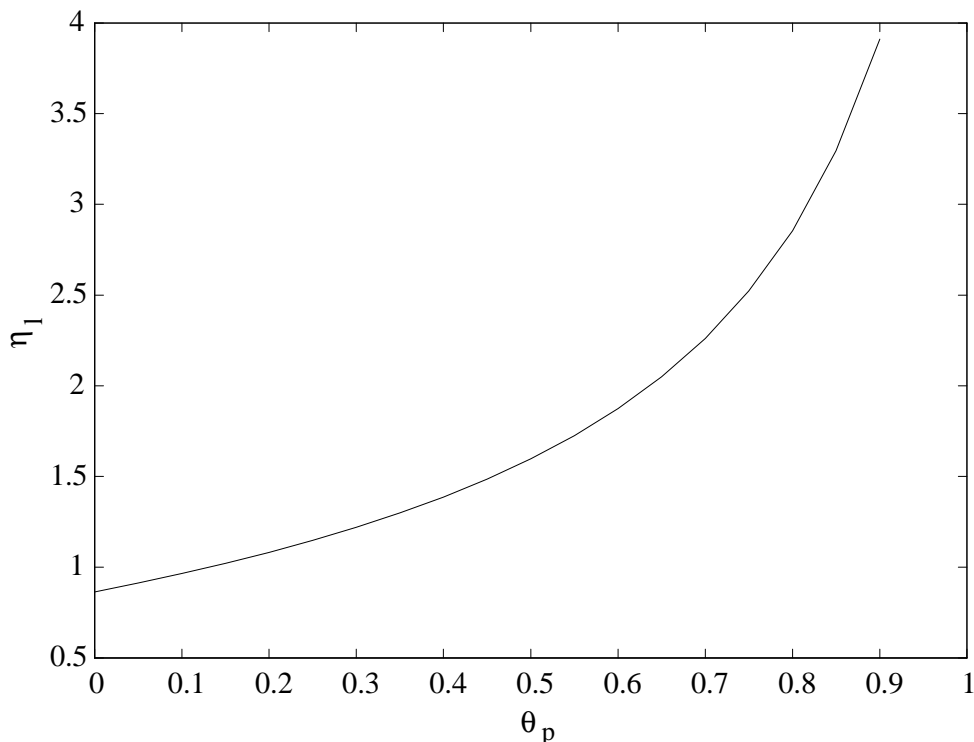


Figure 2: Price stickiness and aggregate smoothness of capital accumulation.

A decrease in the value assigned to parameter θ_p results in a decrease in the smoothness of aggregate capital accumulation, as measured by the associated change in the value of parameter η_l . The intuition is simple. With more flexible prices a firm currently choosing to increase its capital stock is more likely to be able to create additional demand (by decreasing its price) over the expected lifetime of the chosen capital stock. This increases its marginal return to capital and hence an investing firm is more willing to increase its capital stock in response to an increase in the average real return.

Second, the role of monopolistic competition is analyzed under the assumption of perfectly flexible prices.¹⁶ Again, all the remaining parameters are held constant at their baseline values. The results are shown in Figure 3.

¹⁶See Hornstein (1993) for an early analysis of the role of monopolistic competition for equilibrium dynamics in the context of an RBC model featuring increasing returns to scale.

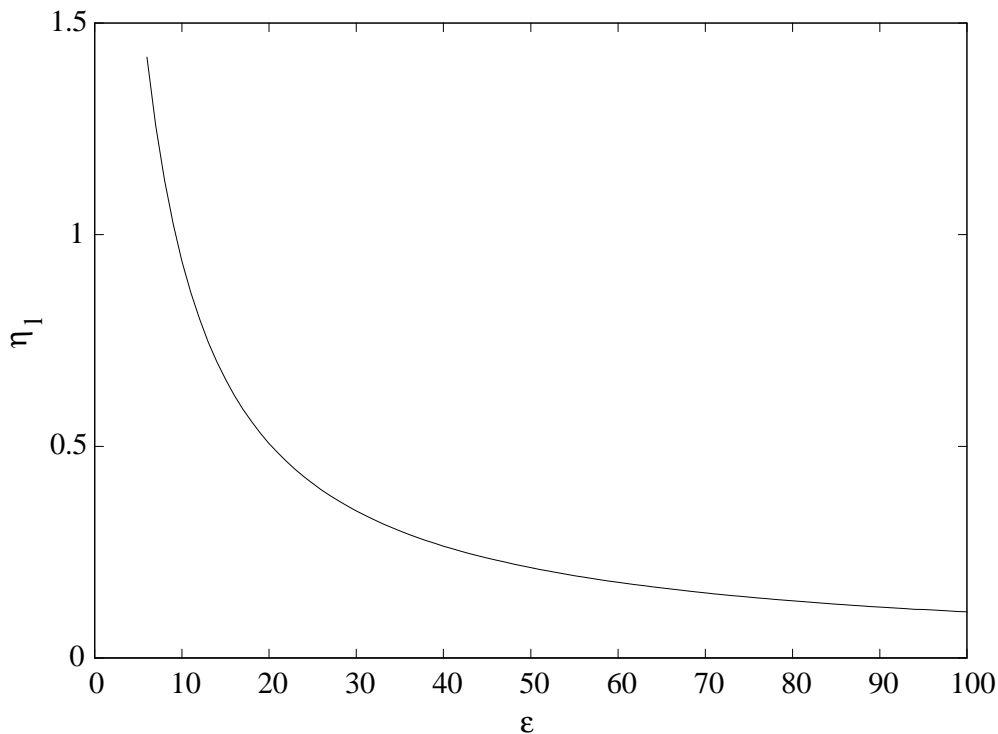


Figure 3: Market power and aggregate smoothness of capital accumulation.

An increase in the value assigned to parameter ε , which is inversely related to the market power of firms, is associated with a decrease in parameter η_l . In a more competitive economy a smaller price change is needed to bring about any given increase in a firm's demand. This makes an investing firm less reluctant to change its capital stock in response to an increase in the average real marginal return to capital.

Finally, the features of price stickiness and monopolistic competition are turned off in our model. In their absence the linearized equilibrium dynamics of our lumpy investment model are identical to the ones implied by a frictionless investment economy. This can be seen by inspecting the reduced form parameter η_l in the flexible

price case. It is given by

$$\eta_l = \frac{\theta_k}{(1 - \theta_k)(1 - \beta\theta_k)} \frac{1 - \beta(1 - \delta)}{1 - \alpha + \varepsilon\alpha}.$$

Clearly, in the limit as $\varepsilon \rightarrow \infty$, η_l approaches zero.

4 Conclusion

Viewed through the lens of an RBC model, plant-level lumpy investment appears to be irrelevant for business cycle dynamics: The implied equilibrium dynamics are almost identical to the ones associated with the alternative assumption of frictionless investment. This is the main result in Thomas (2002). However, in the NK literature it is typically assumed that aggregate capital accumulation is smoother than would be the case if investment was frictionless. Woodford (2005) argues that this assumption is crucial, for otherwise NK models could not account for the dynamic effects of monetary policy shocks. In the present paper the following question is therefore addressed: Can the required smoothness of aggregate capital accumulation be rationalized under the empirically plausible assumption of lumpy plant-level investment? Our answer is yes. In fact, our NK model with lumpy investment is equivalent to its counterpart featuring a convex capital adjustment cost at the level of the production unit. Importantly, the lumpy investment model implies that empirically plausible parameter values result in aggregate smoothness of capital accumulation of the kind which is needed to render NK models capable of explaining the dynamic effects of monetary policy shocks.

In our current framework, firms get an opportunity to invest (and to re-optimize their prices) independently of the deviations of their own capital stocks (and posted prices) from their optimal values. It would therefore be interesting to investigate how the implications of lumpy investment for aggregate dynamics would be affected,

if investment and price-setting was undertaken in a state-contingent fashion. Under those circumstances, investment and price-setting would be conducted mostly by firms whose capital and/or price is most out of line and this in turn might lead to some changes in the quantitative results. This caveat implied by the simplicity of our model notwithstanding, the results in the current paper show that price stickiness and/or the monopoly power of firms can lead to relevance of lumpy investment for the determination of aggregate macroeconomic variables in general equilibrium. We conjecture that these two economic mechanisms will play a quantitatively dominant role in any model featuring forward-looking price-setting and investment. Clearly, this warrants future research.

Appendix A

In order to find the inflation equation and the law of motion of the aggregate capital stock for our lumpy investment model the method developed in Woodford (2005) is employed. First, equation (12) is combined with (13) and equation (14) with (15). Log-linearizing and rearranging the resulting expressions gives

$$\begin{aligned} \widehat{p}_t^*(i) = & E_t^p \left\{ \sum_{j=1}^{\infty} (\beta\theta_p)^j \pi_{t+j} + \frac{(1-\beta\theta_p)(1-\alpha)}{1-\alpha+\varepsilon\alpha} \sum_{j=0}^{\infty} (\beta\theta_p)^j mc_{t+j} \right. \\ & \left. - \frac{(1-\beta\theta_p)\alpha}{1-\alpha+\varepsilon\alpha} \sum_{j=0}^{\infty} (\beta\theta_p)^j \widehat{k}_{t+j}(i) \right\}, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \widehat{k}_{t+1}^*(i) = & E_t^k \left\{ \sum_{j=1}^{\infty} (\beta\theta_k)^j \Delta k_{t+j+1} - (1-\beta\theta_k)\varepsilon \sum_{j=0}^{\infty} (\beta\theta_k)^j \widehat{p}_{t+j+1}(i) \right. \\ & + (1-\alpha)(1-\beta\theta_k) \sum_{j=0}^{\infty} (\beta\theta_k)^j ms_{t+j+1} \\ & \left. - \frac{(1-\alpha)(1-\beta\theta_k)}{1-\beta(1-\delta)} \sum_{j=0}^{\infty} (\beta\theta_k)^j (i_{t+j} - \pi_{t+j+1} - \rho) \right\}, \end{aligned} \quad (\text{A2})$$

where $\widehat{P}_t(i) \equiv \frac{P_t(i)}{P_t}$ and $\widehat{K}_t(i) \equiv \frac{K_t(i)}{K_t}$ denote, respectively, firm i 's relative price and relative to average capital stock, and the definitions $\widehat{P}_t^*(i) \equiv \frac{P_t^*(i)}{P_t}$ and $\widehat{K}_t^*(i) \equiv \frac{K_t^*(i)}{K_t}$ have also been used. Second, rules for price-setting and for investment are posited

$$\widehat{p}_t^*(i) = \widehat{p}_t^* - \tau_1 \widehat{k}_t(i), \quad (\text{A3})$$

$$\widehat{k}_{t+1}^*(i) = \widehat{k}_{t+1}^* - \tau_2 \widehat{p}_t(i), \quad (\text{A4})$$

where τ_1 and τ_2 are unknown parameters and \widehat{P}_t^* and \widehat{K}_{t+1}^* denote, respectively, the average newly set relative price and the average newly chosen relative capital stock. Third, the Bernoulli assumption for the price-setting lottery is used and combined

with the definition of the price index. This results in

$$\pi_t = \frac{1 - \theta_p}{\theta_p} \widehat{p}_t^*. \quad (\text{A5})$$

Fourth, the Bernoulli assumption for the investment lottery is invoked and combined with the definition of aggregate capital, which allows us to write

$$k_{t+1} = k_t + \frac{1 - \theta_k}{\theta_k} \widehat{k}_{t+1}^*. \quad (\text{A6})$$

Hence

$$\begin{bmatrix} \widetilde{E}_t \widehat{p}_{t+1}(i) \\ \widetilde{E}_t \widehat{k}_{t+1}(i) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \widehat{p}_t(i) \\ \widehat{k}_t(i) \end{bmatrix},$$

where

$$\mathbf{A} \equiv \begin{bmatrix} 1 & \tau_1(1 - \theta_p) \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \theta_p & 0 \\ -(1 - \theta_k)\tau_2 & \theta_k \end{bmatrix},$$

and \widetilde{E}_t is meant to indicate that the expected value is taken before the firm gets to know whether or not it will be able to invest in period t . Stability requires that both roots of \mathbf{A} are inside the unit circle. Next, the remaining conditions for the unknown coefficients are determined.

Law of motion of aggregate capital

The price-setting rule (A3) is used to substitute for the term $\sum_{j=0}^{\infty} (\beta\theta_k)^j E_t^k \widehat{p}_{t+j+1}(i)$

in (A2). The result is shown in the next equation

$$\begin{aligned}
\psi \widehat{k}_{t+1}^*(i) &= \psi \sum_{j=1}^{\infty} (\beta \theta_k)^j E_t \{ \Delta k_{t+j+1} \} - \frac{\theta_p (1 - \beta \theta_k) \varepsilon}{1 - \beta \theta_p \theta_k} \widehat{p}_t(i) \\
&+ (1 - \alpha) (1 - \beta \theta_k) \sum_{j=0}^{\infty} (\beta \theta_k)^j E_t \{ m s_{t+j+1} \} \\
&- \frac{(1 - \alpha) (1 - \beta \theta_k)}{1 - \beta (1 - \delta)} \sum_{j=0}^{\infty} (\beta \theta_k)^j E_t (i_{t+j} - \pi_{t+j+1} - \rho), \quad (\text{A7})
\end{aligned}$$

where $\psi \equiv 1 - \frac{\tau_1 (1 - \theta_p) \varepsilon}{1 - \beta \theta_p \theta_k}$. Averaging the last equation over all investing firms and subtracting the resulting equation from (A7) it is possible to write $\widehat{k}_{t+1}^*(i)$ as a function of \widehat{k}_{t+1}^* and $\widehat{p}_t(i)$, as in the investment rule (A4). This allows us to impose the following restriction on parameter τ_2

$$\tau_2 = \frac{\theta_p (1 - \beta \theta_k) \varepsilon}{1 - \beta \theta_p \theta_k - \tau_1 (1 - \theta_p) \varepsilon}. \quad (\text{A8})$$

In order to derive the law of motion of capital, equation (A7) is aggregated over all investing firms and the resulting expression is combined with equation (A6). This allows us to write

$$\Delta k_{t+1} = \beta E_t \{ \Delta k_{t+2} \} + \frac{1}{\eta_t} [(1 - \beta (1 - \delta)) E_t \{ m s_{t+1} \} - (i_t - E_t \{ \pi_{t+1} \} - \rho)], \quad (\text{A9})$$

where $\eta_t^{-1} = \frac{(1 - \theta_k)(1 - \beta \theta_k)}{\theta_k} \frac{1 - \alpha}{1 - \beta (1 - \delta)} \frac{1}{\psi}$.

Inflation equation

The inflation equation is derived in an analogous manner. Combining the log-linearized first-order condition for price-setting (A1) with the investment rule (A4)

gives

$$\begin{aligned} \xi \widehat{p}_t^*(i) &= \xi \sum_{j=1}^{\infty} (\beta \theta_p)^j E_t \{ \pi_{t+j} \} + \frac{(1 - \beta \theta_p)(1 - \alpha)}{1 - \alpha + \varepsilon \alpha} \sum_{j=0}^{\infty} (\beta \theta_p)^j E_t \{ mc_{t+j} \} \\ &\quad - \frac{(1 - \beta \theta_p) \alpha}{1 - \alpha + \varepsilon \alpha} \frac{1}{1 - \beta \theta_p \theta_k} \widehat{k}_t(i), \end{aligned} \quad (\text{A10})$$

where $\xi \equiv 1 - \frac{\alpha(1-\theta_k)\beta\theta_p\tau_2}{(1-\alpha+\varepsilon\alpha)(1-\beta\theta_p\theta_k)}$. Next, the last equation is averaged over all price setters and the resulting expression is subtracted from (A10). After invoking the price-setting rule (A3), the following restriction can be imposed on parameter τ_1

$$\tau_1 = \frac{(1 - \beta \theta_p) \alpha}{(1 - \alpha + \varepsilon \alpha) (1 - \beta \theta_p \theta_k) - \alpha (1 - \theta_k) \beta \theta_p \tau_2}. \quad (\text{A11})$$

Equations (A8) and (A11), when combined with the two stability conditions, determine the two unknown parameters τ_1 and τ_2 . Last, the inflation equation is obtained by averaging (A10) over price-setters and using (A5). This results in

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa_l mc_t, \quad (\text{A12})$$

where $\kappa_l \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\varepsilon\alpha} \frac{1}{\xi}$.

Appendix B

Claim:

Fix η_c and suppose that θ_k is chosen in such a way that we have $\eta_c = \eta_l$, i.e. the two laws of motion of capital implied by the lumpy investment model and the convex adjustment cost benchmark case coincide. This implies $\kappa_l = \kappa_c$, i.e. the two inflation equations implied by the two models are also identical. The two models are therefore observationally equivalent, up to the first order.

Proof:

The strategy of the proof is as follows. First, it is observed that κ_l is equal to κ_c if the respective values of two coefficients in the two models are identical. Second, it is shown that the respective sets of restrictions pinning down these coefficients in the two models are in fact identical, if the assumption of our claim is met. In what follows these two steps are developed.

Step 1

Our starting point is the convex adjustment cost model. In that model parameter κ_c is determined in the following way (see, Sveen and Weinke 2005)

$$\kappa_c = \frac{(1 - \theta_p)(1 - \beta\theta_p)}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon} \frac{1}{\xi_c}, \quad (\text{B1})$$

where $\xi_c \equiv 1 + \frac{\alpha\beta\theta_p\lambda_2}{(1-\alpha+\alpha\varepsilon)(1-\beta\theta_p\lambda_1)}$ and parameters λ_1 and λ_2 are non-linear functions of the structural parameters, as discussed below.¹⁷

¹⁷It should be noted that there is a typo in the definition of parameter κ (which corresponds to κ_c in the present paper) on page 38 in the appendix of Sveen and Weinke (2005). Coefficient ω entering that definition should read $1 + \frac{\alpha\varepsilon}{1-\alpha} + \frac{\alpha\tau_3}{1-\alpha} \frac{\beta\theta}{1-\beta\theta\tau_2}$. The mistake occurred, however, only in the text. All the computations conducted in that paper are correct, to our best knowledge.

The corresponding parameter in the lumpy investment economy, κ_l , is given by:

$$\kappa_l = \frac{(1 - \theta_p)(1 - \beta\theta_p)}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon} \frac{1}{\xi}, \quad (\text{B2})$$

where $\xi \equiv 1 - \frac{\alpha\beta\theta_p(1-\theta_k)\tau_2}{(1-\alpha+\varepsilon\alpha)(1-\beta\theta_p\theta_k)}$. Let κ_1 denote the particular value of the lumpiness parameter which implies that we have $\eta_c = \eta_l$,¹⁸ and also define $\kappa_2 \equiv -(1 - \kappa_1)\tau_2$. For this parameter choice we have $\xi = 1 + \frac{\alpha\beta\theta_p\kappa_2}{(1-\alpha+\varepsilon\alpha)(1-\beta\theta_p\kappa_1)}$, which shows that parameters κ_l and κ_c would take identical values, if we had $\kappa_1 = \lambda_1$ and $\kappa_2 = \lambda_2$.

Step 2

In order to show that we have $\kappa_1 = \lambda_1$ and $\kappa_2 = \lambda_2$, if the assumption of our claim is met, we analyze the respective sets of restrictions which pin down the values of these parameters in the two models. Our starting point is once again the convex adjustment cost case. In that model parameters λ_1 and λ_2 are determined (jointly with parameter φ) by the following three non-linear equations and by two stability conditions, which will be considered later:

$$\varphi = \frac{(1 - \beta\theta_p)\alpha}{(1 - \alpha + \alpha\varepsilon)(1 - \beta\theta_p\lambda_1) + \alpha\beta\theta_p\lambda_2}, \quad (\text{B3})$$

$$\lambda_2 = \frac{\Xi\theta_p\lambda_1}{\beta\theta_p\lambda_1 - 1}, \quad (\text{B4})$$

$$0 = -\Phi\lambda_2 + \beta\lambda_1\lambda_2 + \beta\lambda_2\theta_p - (1 - \theta_p)\varphi(\beta\lambda_2 - \Xi)\lambda_2 - \Xi\theta_p, \quad (\text{B5})$$

where $\Phi \equiv 1 + \beta + \frac{\Xi}{\varepsilon}$ and $\Xi \equiv \frac{[1-\beta(1-\delta)]\varepsilon}{\eta_c(1-\alpha)}$.

Next, the lumpy investment model is analyzed. Parameters κ_1 and κ_2 are determined (jointly with parameter τ_1) by three non-linear equations and by two stability conditions, which will be considered later. Using the assumption of our claim, i.e. setting $\eta_l = \eta_c$, allows us to write these three non-linear equations in the following

¹⁸This value is uniquely pinned down, as we are going to see.

way:

$$\tau_1 = \frac{(1 - \beta\theta_p)\alpha}{(1 - \alpha + \varepsilon\alpha)(1 - \beta\theta_p\kappa_1) + \alpha\beta\theta_p\kappa_2}, \quad (\text{B6})$$

$$\kappa_2 = \frac{(1 - \kappa_1)\theta_p(1 - \beta\kappa_1)\varepsilon}{1 - \beta\theta_p\kappa_1 - \tau_1(1 - \theta_p)\varepsilon}, \quad (\text{B7})$$

$$\eta_c = \frac{\kappa_1}{(1 - \kappa_1)(1 - \beta\kappa_1)} \frac{1 - \beta(1 - \delta)}{(1 - \alpha)} \left[1 - \frac{\tau_1(1 - \theta_p)\varepsilon}{1 - \beta\theta_p\kappa_1} \right]. \quad (\text{B8})$$

Finally, it is shown that parameters φ , λ_1 and λ_2 and parameters τ_1 , κ_1 , and κ_2 are pinned down by two identical sets of restrictions under the assumption of our claim.

First, it is clear that equation (B6) takes the same form as its counterpart (B3).

Second, equation (B8) is rewritten in the following way:

$$\frac{1}{1 - \beta\theta_p\kappa_1 - \tau_1(1 - \theta_p)\varepsilon} = \frac{\kappa_1}{(1 - \kappa_1)(1 - \beta\kappa_1)} \frac{1 - \beta(1 - \delta)}{\eta_c(1 - \alpha)} \frac{1}{1 - \beta\theta_p\kappa_1}.$$

Substituting the last equation into (B7) allows us to write:

$$\kappa_2 = \frac{\Xi\theta_p\kappa_1}{\beta\theta_p\kappa_1 - 1},$$

which takes the same form as restriction (B4) above. Third, (B8) can be written as:

$$\begin{aligned} 0 &= -\varepsilon + \varepsilon\kappa_1 + \varepsilon\beta\kappa_1 - \varepsilon\beta\kappa_1^2 + \varepsilon\beta\theta_p\kappa_1 - \varepsilon\beta\theta_p\kappa_1^2 - \varepsilon\beta^2\theta_p\kappa_1^2 \\ &\quad + \varepsilon\beta^2\theta_p\kappa_1^3 + \kappa_1\Xi - \beta\theta_p\kappa_1^2\Xi - \varepsilon\kappa_1\tau_1\Xi + \varepsilon\theta_p\kappa_1\tau_1\Xi. \end{aligned}$$

Substituting (B4) for λ_2 in equation (B5) and simplifying gives:

$$\begin{aligned} 0 &= -\varepsilon + \varepsilon\lambda_1 + \varepsilon\beta\lambda_1 - \varepsilon\beta\lambda_1^2 + \varepsilon\beta\theta_p\lambda_1 - \varepsilon\beta\theta_p\lambda_1^2 - \varepsilon\beta^2\theta_p\lambda_1^2 \\ &\quad + \varepsilon\beta^2\theta_p\lambda_1^3 + \lambda_1\Xi - \beta\theta_p\lambda_1^2\Xi - \varepsilon\lambda_1\varphi\Xi + \varepsilon\theta_p\lambda_1\varphi\Xi, \end{aligned}$$

which shows that the two restrictions (B8) and (B5) take the same form as well.

It still remains to be shown that the two stability conditions implied by the two models are also identical under the assumption of our claim. But this can easily be seen to be true. Since we know already that the candidate values for the relevant parameters must be identical under our assumption, i.e. $\kappa_1 = \lambda_1$, $\kappa_2 = \lambda_2$ and $\varphi = \tau_1$ must hold for each candidate, we might rewrite matrix \mathbf{A} in the appendix in the following way:

$$\mathbf{A} \equiv \begin{bmatrix} 1 & \varphi(1 - \theta_p) \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \theta_p & 0 \\ \lambda_2 & \lambda_1 \end{bmatrix}.$$

But this is exactly the matrix, which is used to check stability in the convex adjustment cost model. Under the assumption of our claim the two sets of conditions determining the relevant coefficients are therefore identical in the two models.

References

- Bachmann, Ruediger, Ricardo J. Caballero, and Eduardo M.R.A Engel (2006): “Lumpy Investment in Dynamic General Equilibrium”, NBER Working Paper 12336.
- Caballero, Ricardo J. (1999): “Aggregate Investment”, In: Taylor J., Woodford M. (Eds.) *Handbook of Macroeconomics*, North Holland, Amsterdam, 813-862.
- Caballero, Ricardo J., and Eduardo M.R.A Engel (1999): “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach”, *Econometrica* 67, 783-826.
- Calvo, Guillermo (1983): “Staggered Prices in a Utility Maximizing Framework”, *Journal of Monetary Economics* 12, 383-398.
- Casares, Miguel, and Bennett T. McCallum (2000): “An Optimizing IS-LM Framework with Endogenous Investment”, NBER Working Paper 7908.
- Christiano, Lawrence J., Martin S. Eichenbaum, and Charles Evans (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *Journal of Political Economy* 113, 1-45.
- Doms, Mark, and Timothy Dunne (1998): “Capital Adjustment Patterns in Manufacturing Plants”, *Review of Economic Dynamics* 1, 409-429.
- Galí, Jordi, Mark Gertler, and David López-Salido (2001): “European Inflation Dynamics”, *European Economic Review* 45, 1237-1270.
- Gourio, François, and Anil K. Kashyap (2007): “Investment Spikes: New Facts and a General Equilibrium Exploration”, forthcoming *Journal of Monetary Economics*.
- Hornstein, Andreas (1993): “Monopolistic Competition, Increasing Returns to Scale, and the Importance of Productivity Shocks”, *Journal of Monetary Economics* 31, 299-316.

- Khan, Aubhik, and Julia K. Thomas (2004): “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics”, Federal Reserve Bank of Philadelphia Working Paper No. 04-15.
- Kiyotaki, Nobuhiro, and John Moore (1997): “Credit Cycles”, *Journal of Political Economy* 105, 211-248.
- Rotemberg, Julio J. (1987): “The New Keynesian Microfoundations”, In: Fischer, S. (Ed.) *NBER Macroeconomics Annual 1987*, MIT Press, Cambridge, MA, 69-104.
- Sbordone, Argia M. (2002): “Prices and Unit Labor Costs: A New Test of Price Stickiness”, *Journal of Monetary Economics* 49, 265-292.
- Smets, Frank, and Raf Wouters (2003): “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”, *Journal of the European Economic Association* 1, 1123–1175.
- Svein, Tommy, and Lutz Weinke (2005): “New Perspectives on Capital, Sticky Prices, and the Taylor Principle”, *Journal of Economic Theory* 123(1), 21–39.
- Thomas, Julia K. (2002): “Is Lumpy Investment Relevant for the Business Cycle?”, *Journal of Political Economy* 110(3), 508-534.
- Veracierto, Marcelo L. (2002): “Plant-Level Irreversible Investment and Equilibrium Business Cycles”, *American Economic Review* 92(1), 181-197.
- Woodford, Michael (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, NJ.
- Woodford, Michael (2005): “Firm-Specific Capital and the New Keynesian Phillips Curve”, *International Journal of Central Banking* 2, 1-46.