Risk Taking in Selection Contests*

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Abstract

We study selection contests in which the strategic variable is degree of risk rather than amount of effort. The selection efficiency of such contests is examined. We show that the selection efficiency of a contest may be improved by limiting the competition in two ways; a) by having a small number of contestants, and b) by restricting contestant quality. The results may contribute to our understanding of such diverse phenomena as promotion processes in firms, selection of fund managers and research tournaments.

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1 Introduction

In a contest or a tournament, rewards are based on the relative performance of the contestants. Contests serve two different purposes. First, tournaments among workers can mitigate incentive problems when the effort of workers is unobservable. Second, tournaments serve as a selection mechanism. For example, since employers do not necessarily know which workers are the most able, promotions are often based on a comparison of the observed productivity of the workers; the firm promotes the top-ranked worker.

In this paper we focus on the selection aspect of contests, in the case where risk taking is the strategic variable of the contestants. Employees involved in a promotion process or tenure process, for example, may choose tasks that differ in risk profile to show off their abilities.[1] Another example is fund managers' competition for new investors. Empirical studies show that fund managers with the highest rate of return one year capture the lion's share of subsequent years' investments. Furthermore, these studies show that competition for prospective investments has impact on fund managers' risk taking.[2] Consequently, workers competing for promotion and fund managers competing for the flow of investors may be viewed as taking part in a selection contest in which risk taking is an important strategic variable.

We investigate the selection efficiency of contests in which the contestants optimize their choice of risk, given the risk taking of others. Who will come out on top, bad types or good types? In what way will the selection efficiency depend on, for example, the quality of the contestant pool? We view answering such questions as important to understanding

[1] Or even simpler, the task may be fixed but employees choose between a 'safe' working method (e.g., working thoroughly) and a 'risky' working method (e.g., working hastily).

[2] For example see Chevalier and Ellison (1997) and Brown et al. (1996).

the efficiency of promotion processes in large firms and the efficiency of fund manager selection in financial markets.

Although the case where agents choose both risk and effort seems realistic for many applications, for tractability we confine ourselves to the case where risk taking is the only strategic variable. Moreover, we restrict ourselves to the case where there is only one prize to be won. From that this starting point, we investigate the selection efficiency of contests along two dimensions: the number of contestants and the quality of the pool of contestants. Two natural conjectures are the following: Selection efficiency improves with the quality of the contestant pool, and selection efficiency improves with the number of contestants. Tougher competition makes tougher winners. Our two main results are negative; we show that, in our simple model, neither conjectures necessarily holds true.

The model we work with has two types of agents, a *low* type and a *high* type, each with two possible pure strategies, $safe$ and $risky$. The $risky$ strategy induces a (not necessarily mean preserving) spread in the probability distribution of individual output compared to the $safe$ strategy. For a given risk level, the *high* type has a higher expected output than the *low* type. The output space is discrete. The latter assumption is fairly restrictive, and in Appendix A we use numerical techniques to show that the main results from the discrete model also apply in continuous models.

We focus on what seems to be the most natural measure of selection efficiency of a contest; the probability of a *high* type agent winning it. We denote this probability by Π. We show that Π may decrease with a pool of agents of higher quality, i.e., an increase in the share of *high* ability agents in the pool. To see the underlying intuition, notice that increasing the quality of the pool has two effects. The first is the statistical effect: a higher

quality of the pool increases Π, holding the strategies of the types fixed. The second effect is the equilibrium effect: increasing the quality of the pool shifts the equilibrium of the game to one with increased risk taking. The latter effect may decrease Π. Thus we show that the statistical effect's positive influence on Π may be dominated by the equilibrium effect's negative influence on Π. A surprising implication is that a firm may discriminate against agents who are likely to be highly skilled by not allowing them to take part in the contest.[3]

A similar intuition can be applied to our discussion of the effect on Π of increasing the number of contestants. Suppose that the number of contestants increases. In that case, the probability of a *high* type agent being included in the contest obviously increases (a positive statistical effect). However, increasing the number of contestants also implies more risk taking in equilibrium (the equilibrium effect), which may harm to selection efficiency. We show that the positive statistical effect of increasing the number of contestants may be weaker than the negative equilibrium effect. Thus a firm may improve selection efficiency by limiting competition for higher-rank positions.

Although it has often been argued that contests serve both motivation and selection functions (see e.g., Lazear and Rosen (1981), Schlicht (1988)), the tournament literature has mostly focused on the case with homogenous agents, where selection problems in the sense discussed here do not arise.[4] Papers that do consider the case with heterogeneous agents restrict the discussion to how a tournament reward structure may motivate agents to

[3] Baye et al. (1993) reports a related exclusion result in a complete information setup for all-pay auctions. Auction revenue may increase if agents with high valuations are excluded.

[4] In the case with homogenous agents and effort as a strategic variable, Nti (1997) showed that increasing the number of workers competing for a prize may result in a decline in the overall level of effort. Thus our results on the gains from limiting competition has its counterpart in the received literature.

work hard. An exception is Rosen (1986) (section V), which considers both the motivation function and the selection function of contests. The present paper complements Rosen (1986) in considering selection efficiency under risk taking instead of under "effort taking". Also, since Rosen confines attention to the case where there is no private information about own type, our aim is, in that sense, broader in scope.

Harrington (1999, 1998) consider a promotion game where agents with the highest output are promoted to a higher level in an organization. If agents are endowed with simple behavior rules, Harrington (1998) shows that agents that are unresponsive to changes in the environment reach the top of the organization. Harrington (1999), on the other hand, allows agents to act strategically and shows that the "rigidity" result of Harrington (1998) can be reversed. While Harrington (1998) does not consider strategic actions and Harrington (1999) assumes that agents are homogenous, the present paper considers heterogenous agents that act strategically.[5]

The efficiency of various selection procedures is a main topic in the statistical decision theory (see e.g., Gibbons et al. (1977)). By focusing on selection efficiency as the measure of the success of a contest, instead of e.g., aggregate output, our work is in that sense closer to statistical decision theory than to the tournament literature. In contrast to the statistical decision theory, the present paper considers the selection efficiency of a contest when agents act strategically. The strategic element makes the noise in the selection process we study

[5] Using tools from evolutionary game theory Dekel and Scotchmer (1999) find an evolutionary pressure towards risk loving preferences provided that those who breed in a population is determined by a contest (and where a child inherits the risk preferences of its parents). The focus of Dekel and Scotchmer (1999) is very different from our focus (there is e.g., no discussion of selection efficiency in Dekel and Scotchmer (1999)), but the models applied are similar.

A patent race is a kind of contest in which there is only one prize – the patent. Risk taking in such contests has been carefully analyzed in e.g. Klette and de Meza (1986), Cabral (1997), and Dasgupta and Maskin (1987). However, selection issues do not arise in these papers – only the date of innovation matters.

endogenous, while the noise in the selection processes studied by statistical decision theory is exogenous. Thus, the statistical decision theory literature only considers statistical effects, while we consider the interaction between statistical effects and equilibrium effects.

The remainder of the paper is organized as follows. In Part 2 we describe the model. In Part 3 we performe the ananlysis. Part 4 concludes. In Appendix A we use numerical techniques to see whether our basic insights from Part 3 are robust to making the model more continuous. All proofs are relegated to Appendix B.

2 The Model

Consider a setting in which a principal arranges a contest in order to identify a talented agent. We assume that the principal can only observe the rank of the agents, and awards a prize to the agent with the highest rank, or ouput.[6] There are n risk-neutral agents competing for the prize, whose value is normalized to 1.[7] The individual output space Z is finite and consists of four elements; $Z := \{z_1, z_2, z_3, z_4\}$, where $z_1 < z_2 < z_3 < z_4$ (tied winners have an equal chance of obtaining the prize). There are two types of agents, *low* and *high*, with θ denoting the share of the *high* type in the pool from which the n agents are drawn. Both types have an opportunity cost of participation equal to zero, and hence

[6] As pointed by e.g., ..., cases where the principal mainly has ordinal information on individual output, or where only ordinal information is verifiable (Malcomson (1984)), are common in practice. If cardinal information on individual output is available and verifiable, an interesting question, that goes beyond the aims of the present paper, is whether such cardinal information can make schemes where the prize goes to an agent with an output in the 'middle' optimal. (Notice that such non-monotonic schemes have the weakness that they give agents incentives to dispose with parts of their output in equilibrium. For example, fund managers have an incentive to inflate trading costs.)

[7] As there are no costs associated with risk taking in our model, the size of the prize has no effect on equilibria. Notice that the freedom with respect to the size of the prize makes the model consistent with any degree of bargaining power between the firm and the contestants.

the group of contestants is a true random sample from the pool.[8] Agents of each type have two pure strategies, $safe$ and $risky$. If a low type agent chooses $safe$ then her output is z_2 with certainty. If a $high$ agent chooses $safe$ then her output is z_3 with certainty. If a low type agent chooses $risky$ then her output is z_1 with probability $1-x$, and z_4 with probability x. If a $high$ type agent plays $risky$ then her output is z_1 with probability $1-y$, and z_4 with probability y, where $y > x$. We do not exclude mixed strategies, and thus the (mixed) strategy space has the usual continuity properties. Outputs are assumed to be statistically independent. Expected utility for an agent equals her win probability, since we assume that there are no costs associated with risk taking. Alternative approaches are discussed in a footnote.[9]

A special case of the model is the case where expected output is constant across projects of a given type, i.e., the case when the distribution of output under the $risky$ strategy is a mean preserving spread (MPS) of the distribution of output under the $safe$ strategy. In the numerical analysis we explicitly assume that risky strategies induce a MPS of the distribution of output under the $safe$ strategy. Notice, however, that the model is not restricted to the MPS case.

[8] A model with self-selection into different contests (in the spirit of Bhattacharya and Guasch (1988)) is a possible extension of the present work. For example, it might be possible to construct a pair of contests (with different degrees of possible risk taking), in which the *low* (*high*) type individuals self-select into the contest with the high (low) level of potential risk taking.

[9] Our model is a straightforward multi-type extension of the models in Lambert (1986) and Diamond (1998) (who study a single agent principal agent problem). We decided to use this model after doing several attempts on other, presumably richer, models. Let us give an example. A natural formulation is to let individual output be normally distributed with fixed mean (interpreted as type) and endogenous variance (risk taking). The unrestricted version of this model (no costs or limits to increasing variance) gives the unsatisfactory conclusion that the low type's variance approaches infinity, securing a $\frac{1}{n}$ chance of winning, regardless of the action of the high type. Thus the outcome of the contest is random in a strict sense, i.e., $\Pi = \theta$. Recall that an agent's type is simply his mean and notice that this conclusion holds for any finite distance between the high type and the low type. Less obviously, the result holds for any number of types. These annoying results can be avoided by assuming a (possibly U-shaped) cost to risk taking. Unfortunately, we found such models too difficult to solve analytically, except cases with very restrictive assumptions about the cost function.

3 Equilibrium Analysis

We now consider the incomplete information game $\Gamma(n, \theta)$, where an agent does not know the type of the other contestants, but she knows n and θ and her own type. A strategy is a mapping from the type space T, where $T := \{low, high\}$, to the action space C, where $C := \{safe, risky\}$. We denote the set of symmetric pure strategies S, where $S := \{(safe, safe),\ (safe, risky),\ (risky, safe),\ (risky, risky)\}$, with the *low* type's action written first. The key endogenous variable is the probability of a *high* type agent winning the prize, denoted $\Pi(\Gamma)$.[10] We confine our attention to (symmetric) Bayes-Nash equilibria (BNE), i.e., strategy tuples where all agents maximize their probability of winning given the strategy of the other agents, and where all agents of the same type play the same strategy.

3.1 Quality of Contestant Pool

To see the effect of increasing the quality of the contestant pool, we consider the case of $n = 2$.[11] Straightforward calculations reveal that there are unique equilibria, and moreover that all four elements of S can be equilibrium strategies depending on the values of the parameters (θ, x, y).[12]

Proposition 1 *All four pure strategy combinations are possible symmetric BNE of* $\Gamma(2, \theta)$. *Furthermore, if there exists a symmetric pure strategy BNE, then it is unique.*

[10] In the case of multiple equilibria, Π depends not only on n and on θ, but also on which equilibrium is being played.

[11] Analogous results can easily be verified for $n = 3$. For increasing n, it becomes increasingly difficult to solve explicitly the polynomial equations that characterize equilibria, since the polynomials are of the order n.

[12] The win probabilities for the different pure strategy combinations are given in the Appendix.

Proof. See Appendix B.

Figure 1 illustrates equilibria for varying x and y combinations given $n = 2$ and $\theta = \frac{1}{2}$.

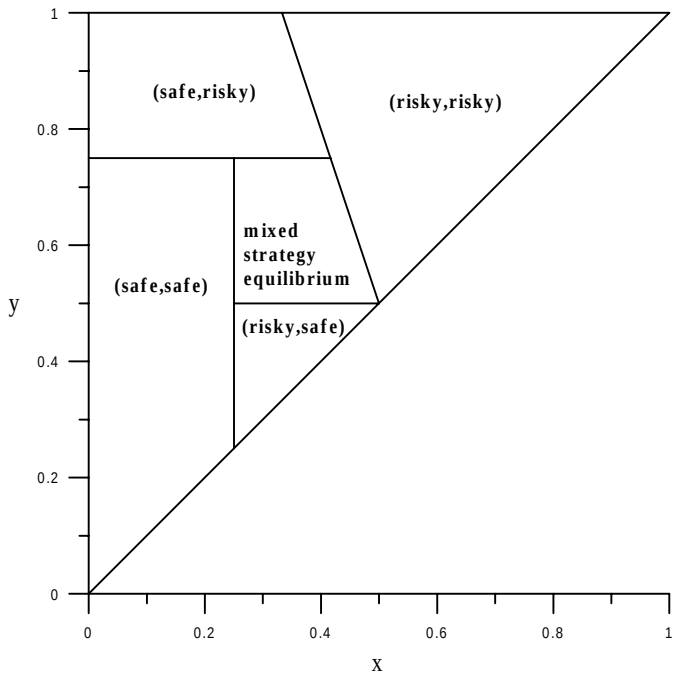

Figure 1: Equilibria for $\theta = \frac{1}{2}$.

Recall that $x(y)$ is the probability of a $low(high)$ agent obtaining the highest outcome if she plays $risky$. With both x and y large, $(risky, risky)$ is the equilibrium, which is natural. In the case where both x and y are small, $(safe, safe)$ is the equilibrium. That seems counterintuitive since in that equilibrium a low agent loses with certainty if the other agent is a $high$ type. The intuition behind the $(safe, safe)$ equilibrium is that the probability of a low type winning against a $high$ type (by playing $risky$) is sufficiently small for the low type to rather care about her best chance of winning were she to play against another low type agent.[13]

[13] Of course, this equilibrium disappears as θ goes to zero.

In some cases it is possible to improve the average ability of the pool of contestants. For example, a firm can hire entry level employees from an Ivy League University rather than from a Minor League University, or an investor can use a professional evaluation firm in order to hire more highly skilled fund managers.[14] A first guess might be that it is advantageous to improve the expected ability of the contestants (i.e. to increase θ), as long as there are no intrinsic costs associated with doing it. However, Proposition 2 shows that this conjecture can be false if increased ability among the contestants induces more risk-taking.

Proposition 2 *Limited Contestant Quality.* Π *may decrease as* θ *increases.*

Proof. See Appendix B.

Π increases with the probability that an arbitrary contestant is of the *high* ability type if we keep the amount of risk taking fixed. This is the statistical effect. However, there is the equilibrium effect also: an increase in θ may result in a shift to an equilibrium with more risk taking, and consequently create more noise in the selection process. This may reduce selection efficiency. The statistical effect may be dominated by the equilibrium effect, and thus Π may increase with a decrease in θ.

When is the equilibrium effect likely to outset the statistical effect? First, an increase in θ may increase the risk taking of the low type, and hence introduce more noise in the selection process and thereby reduce Π. However, Π may also decrease in θ in cases where there is no effect on the low type's equilibrium strategy from increasing θ (i.e., when only the high type plays a more risky strategy after θ is increased). The intuition is that a

[14] Almost all large investors pay professional firms to evaluate mutual fund managers (Heinkel and Stoughton (1994)).

high type agent ignores the negative externality imposed on other high types' probability of winning by choosing a riskier strategy.

In cases in which expected output depends on the risk of the project (i.e., the non-MPS case), selection efficiency as well as aggregate output may be of importance for a principal. Our analysis can straightforwardly be extended to analyze the trade off between aggregate output and selection efficiency. Furthermore, examples in which both selection efficiency and aggregate output decrease in θ can easily be constructed. Hence our non-monotonicity result is robust to making the principal's preferences more general.

3.2 Number of Contestants

To improve Π, it seems natural to increase the number of contestants in order to increase the probability of a good agent participating. For example, if an investor is uncertain about the investment skill of various potential mutual fund managers, it might be tempting to invite a large number to engage in themanagement of its investment portfolio. However, Proposition 3 shows that increased competition, in the sense of increasing the number of contestants, can be a two-edged sword, because increased competition may alter the amount of risk taking in equilibrium.

Proposition 3 *Limited Competition.* Π *may decrease when the number of contestants increases from 2 to 3.*

Proof. See Appendix B.

Proposition 3 shows that the increase in noise may in fact harm the selection process more than the benefits of the greater likelihood of having at least one *high* ability agent

participating in the contest. The equilibrium effect may dominate the statistical effect.[15]

Note also that if a switch from a safe to a risky strategy yields a sufficiently large reduction in expected output, an increase in the number of contestants (which induce more risk taking) may reduce expected aggregated output.

When the number of agents is already large, then adding a player presumably has no equilibrium effect since both types play risky already. An intuition therefore goes that although Π may be decreasing for a small increase in n, Π must increase for a large increase in n. In other words, although an intermediate number of contestants may be worse than a few, a very large number of contestants must be better than a few.[16] But, as Proposition 4 shows, this intuition is false. The proposition builds on a very useful result from Dekel and Scotchmer (1999).

Proposition 4 Π *may be larger for 2 contestants than for an infinite number of contestants.*

Proof. See Appendix B.

4 Conclusion

Contests are used both to induce to work hard and to solve selection problems. It is therefore surprising that the tournament literature has almost exclusively considered the

[15] Notice that in contrast to the case of increasing θ, the statistical effect on Π of increasing n is ambiguous. To see why, assume that the $(risky, safe)$ equilibrium is played for some n. Then, keeping the strategies fixed, Π clearly approaches zero as n increases, and thus the statistical effect is negative for the $(risky, safe)$ equilibrium. On the other hand, the statistical effect on Π of increasing n, given the $(safe, risky)$ equilibrium, is clearly positive. Thus the statistical effect on Π of increasing n is ambiguous, since it depends on the equilibrium strategies played.

[16] Notice that this intuition holds for the quality of contestants. A very high contestant pool quality (θ close to 1) certainly gives at least as good value of Π as low values of θ.

former function. In this paper, however, we have mainly considered how well contests select talented agents, when risk taking is the decision variable of the agents.

We have used promotion decisions in firms and the selection of mutual fund managers as examples of situations where fiercer competition may lead to more risk taking and reduced selection efficiency. However, the insights from our analysis can be applied to other contexts also. For instance, governments and private firms often sponsor tournaments to induce research on specific topics. The reward structure and selection issues of these tournaments is close to what we have discussed in this paper: there is usually only one large prize and selection of a high-quality firm is essential since the winner is going to take care of prospective production.[17] In such tournaments, the participants can usually vary the risk profile of their research strategies. Our results indicate that an organizer of a research tournament may want to restrict the number and quality of contestants in a research tournament.

Taylor (1995) considers how a sponsor of a research contest should induce a high level of effort from the participants – the riskiness of their research strategies is not considered. Nor does Taylor (1995) take into account the fact that the sponsor commonly continues the relationship with the winner through a production contract, and consequently Taylor ignores selection efficiency. Our discussion of selection efficiency under risk taking can be considered as a natural extension of the discussion of effort taking in Taylor (1995).[18]

[17] The prizes are large procurement contracts and/or prize money. Rogerson (1989) used stock-market data to estimate the size of the prize implicit in each production contract awarded after the 12 major aerospace research contests held by the US Department of Defence between 1964 and 1977. He showed that the average award was in the interval 10.2 to 14.6 percent of the market value of an average contestant firm.

[18] Our paper is also related to that of Fullerton and McAfee (1999) which considers the use of auctions for selecting highly qualified contestants for research tournaments. Neither Fullerton and McAfee (1999) nor Taylor (1995) takes into account that firms often can choose among research strategies with varying degree of riskiness.

We have two main results. We show that although increasing the number of firms participating in a contest makes it more likely that the pool of contestants includes a high-quality firm, it might make it less likely that a high-quality firm will be awarded the prize. We also show that an increase in the expected ability or quality of the contestants may make it less likely that a high-quality firm will be selected. The intuition behind the results is that a more competitive tournament – more contestants or higher expected abilities among the contestants – induces firms to adopt riskier strategies, which may harm the selection of high-quality firms. Riskier projects create more noise in the selection contest, and thereby reduce the informativeness of the rank.

Our results provide an explanation of why it seems to be increasingly difficult to identify mutual fund managers with superior investment skills.[19] As the mutual fund market becomes more competitive, fund companies may become more inclined to apply investment strategies (more risk taking) which reduce the investors ability to identify highly skilled fund managers.[20]

A Numerical Analysis

The discrete output space, $\{z_1, z_2, z_3, z_4\}$, places tight restrictions on the type of risk taking allowed. Specifically, the only way for an agent to increase risk is by putting more probability weight on the endpoints z_1 and z_4. With a continuous output space, say the interval $[z_1, z_4]$, increased risk does not necessarily imply more weight at the endpoints. In this appendix we use simulation techniques to consider the case with a continuous output

[19] See e.g. Brown and Goetzmann (1995), Carhart (1997), or Hendricks et al. (1993) for empirical analyses of performance persistence in the mutual fund industry.

[20] The famous failure of Long-Term Capital Management (LTCM) may be an example of a company adopting excessive risky strategies to attract investors.

space and three different ability levels: High (H), Medium (M), and Low (L). The results of this section show that our main results also hold when the output space is continuous.[21]

As before, the agents maximize the probability of being selected by choosing between safe and risky projects. To conduct the simulation analysis we make the following assumptions.

1. The outcomes of the agents' projects are normally distributed with expected outcomes $L = 0$, $M = 3$ or $H = 6$.

2. The agents choose between a safe and a risky project with the same expected outcome. The safe project is assumed to have a standard deviation of 1. The risky project has a standard deviation of σ, where $\sigma \in [3, 7]$.

3. The probability of being of a particular type is:

	L	M	H
Probability	$\frac{1}{2} - \theta$	$\frac{1}{2}$	θ

   An increase in θ implies that it is more likely for any agent to meet an opponent with high ability.

A.1 Quality of the Contestants

In this section we show that Π may decrease with an increase in the quality of the contestants (θ).

[21] The MapleV programs used in this section can be obtained from the authors. We have experimented with different parameter values and obtained similar results, so the results seem robust.

Consider the case with two contestants. It is simple to verify that there exists an equilibrium in dominant strategies where the H type always chooses a safe strategy and the L type always chooses a risky strategy.[22] Let us now focus on the M type. If θ is small, then the likelihood of facing a better contestant is small and the M type behaves as if she is best and, hence, chooses the safe strategy. But if θ is high then the M type is more likely to face a better contestant and, hence, chooses the risky strategy. In Figure 2, the curve G shows the critical values for θ, such that the M type is indifferent between choosing a safe and a risky strategy.

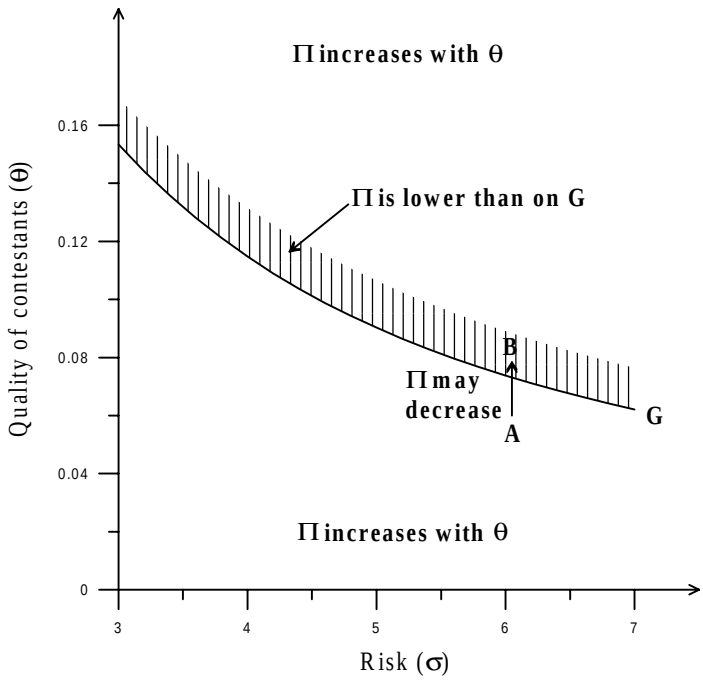

Figure 2: Higher quality (θ) of contestants

The shaded area represents the possibility that an increase in θ reduces Π. Moving

[22] To see why, first note that for type L the high risk strategy dominates the low risk strategy. If she is facing a better type, she will always increase her probability of winning by choosing the riskier strategy. If she is facing another L type she is indifferent about the choice between a high and low risk strategy. Hence, a high risk strategy is a dominant strategy for the L type. Second, note that the low risk strategy is the dominant strategy for the H type. A high risk strategy will increase the probability of low outputs and hence increase the likelihood of less able contestants achieving a higher output. Furthermore, the H type will be indifferent to the choice between low and high risk strategy facing another H type.

northwards from a point on the G line into the shaded area, causes a decrease in Π. To illustrate further, take two points on the diagram and label them A and B. Then Π increases from A to B if B lies further north than A, as long as we do not cross the G line. If A is below the G line and B is above, as illustrated in Figure 2, then Π may decrease.

An increase in the quality of the contestants makes it more likely that one of the contestants is a H type. But higher quality induce the M types to choose a risky strategy, which may decrease Π.

A.2 Number of Contestants

In this section we illustrate that Π may decrease as a result of adding one contestant to a group of two contestants. For simplicity, we focus on the case in which adding a contestant induces the M type to change strategy, but not the L type or the H type. It is straightforward to show that $(risky, risky, safe)_{n=3}$ is a unique equilibrium for $\theta < \frac{1}{5}$, which is the case we consider in the following figure.

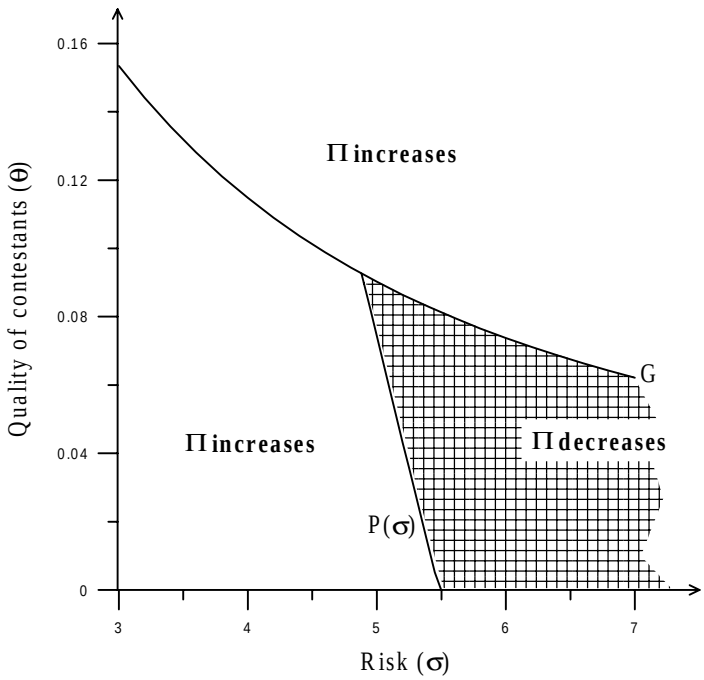

Figure 3: Adding one more contestant

In Figure 3, the line P gives the points where Π is identical for $n = 2$ and $n = 3$. In the shaded area of Figure 3, Π decreases when the number of contestants increases from two to three.

B Proofs

For the sake of brevity, we write s instead of $safe$, r instead of $risky$, l instead of low and h instead of $high$ throughout this appendix.

Proof of Proposition 1: We use the following convention: $U_i(j, k)$ denotes the win probability of an agent of type i when agents of her own type (including herself) play strategy j and agents of the other type play strategy k. For example, $U_H(s, r)$ denotes the win probability of an h agent when all h agents (including herself) play s, and all l agents

play r. The individual payoffs in the symmetric tuples (when all agents of the same type choose the same strategies) are:

$$U_H(r,r) = \tfrac{1}{2}(1+(1-\theta)(y-x)) \quad U_L(r,r) = \tfrac{1}{2}(1+\theta x-\theta y)$$

$$U_H(s,r) = 1-\tfrac{1}{2}\theta - x + \theta x \quad U_L(s,r) = \tfrac{1}{2}(1+\theta)-\theta y$$

$$U_H(r,s) = \tfrac{1}{2}\theta + (1-\theta)y \quad U_L(r,s) = \tfrac{1}{2}(1-\theta)+x\theta$$

$$U_H(s,s) = 1-\tfrac{1}{2}\theta \quad U_L(s,s) = \tfrac{1}{2}(1-\theta)$$

For individual deviations, we use the following convention: $U_i'(j,k)$ denotes the win probability of an agent of type i when she plays strategy $-j$, other agents of her own type play strategy j, and agents of the other type play strategy k. Since the payoff from letting $-j$ be a mixed strategy is a convex combination of playing s and playing r, we only need to consider pure-strategy deviations . For example, $U_H'(s,r)$ denotes the win probability of an h agent playing r, when all other h agents play s, and all l agents play r. The individual payoffs from individual deviation are:

$$U_H'(r,r) = \theta(1-y)+(1-\theta)(1-x)$$

$$U_H'(s,r) = \theta y + (1-\theta)(\tfrac{1}{2}xy + y(1-x) + \tfrac{1}{2}(1-x)(1-y))$$

$$U_H'(r,s) = \theta(1-y)+(1-\theta)$$

$$U_H'(s,s) = y$$

$$U_L'(r,r) = \theta(1-y)+(1-\theta)(1-x)$$

$$U_L'(s,r) = \theta x + (1-\theta)(\tfrac{1}{2}xy + x(1-y) + \tfrac{1}{2}(1-x)(1-y))$$

$$U_L'(r,s) = (1-\theta)(1-x)$$

$$U_L'(s,s) = x$$

Consider equilibrium (r,r). Notice that the payoff from individual deviation is the same for an h agent and an l agent, and moreover that $U_H(r,r) > U_L(r,r)$. Thus we

only have to check a deviation from an l agent*:* if an l agent would not deviate, then an h agent would not deviate. An l agent follows the supposed equilibrium strategy if $\frac{1}{2}(1-\theta y+\theta x) > \theta(1-y)+(1-\theta)(1-x)$, which implies that $y > \frac{1+\theta x-2x}{\theta}$.

Consider equilibrium (s,s). An l agent follows the supposed equilibrium strategy if $x < \frac{1}{2}(1-\theta)$. The condition for an h agent is $y < 1-\frac{1}{2}\theta$.

Consider equilibrium (r,s). An l agent follows the supposed equilibrium strategy if $\frac{1}{2}(1-\theta)+x\theta > (1-\theta)(1-x)$, which implies that $x > \frac{1}{2}(1-\theta)$. The condition for the h type is $\frac{1}{2}(1+(1-\theta)(y-x)) > \theta y+(1-\theta)(\frac{1}{2}xy+y(1-x)+\frac{1}{2}(1-x)(1-y))$, which implies that $y < \frac{1}{2}$.

Consider equilibrium (s,r). An l agent sticks if $\frac{1}{2}(1+\theta)-\theta y > \theta x+(1-\theta)(\frac{1}{2}xy+x(1-y)+\frac{1}{2}(1-x)(1-y))$, which implies that $x < \frac{y+2\theta-3\theta y}{1+\theta}$. The condition for the h type is $\frac{1}{2}\theta+(1-\theta)y > \theta(1-y)+(1-\theta)$, which implies that $y > 1-\frac{1}{2}\theta$.

The uniqueness of BNE, given (x,y), follows directly from the argument. ■

Proof of Proposition 2: There are several areas in the (x,y) diagram where Π decreases with θ. Consider one example. Suppose $x=\frac{1}{5}$ and $y=\frac{1}{4}$. If $\theta=\frac{1}{2}$, then (s,s) is the unique BNE, which gives Π equal to $\frac{3}{4}$. Now increase θ to $\frac{3}{5}$. In that case (r,s) is the unique BNE, and Π equals $\frac{93}{125} < \frac{3}{4}$. Thus we have demonstrated that for $x=\frac{1}{5}$ and $y=\frac{1}{4}$, Π is larger for $\theta=\frac{1}{2}$ than for $\theta=\frac{3}{5}$. ■

Proof of Proposition 3: Consider an example. Let $\theta=\frac{1}{2}$, $x=\frac{1}{5}$, $y=\frac{1}{4}$. First consider the case n=2. Then, from Proposition 2, (s,s) is the unique BNE. That gives $\Pi(2,\frac{1}{2})=\theta^2+2\theta(1-\theta)=\frac{3}{4}=\frac{150}{200}$. Now increase n to 3. In that case, (s,s) is no longer

a BNE since

$$U_L(s,s) = \frac{1}{3}(1-\frac{1}{2})^2 = \frac{1}{12} < U'_L(s,s) = \frac{1}{5}$$

However, (r,s) is indeed the BNE since a)$U_L(r,s) = \frac{67}{300} > U'_L(r,s) = \frac{48}{300}$.While on the other hand, b) $U_H(s,r) = \frac{532}{1200} > U'_H(s,r) = \frac{319}{1200}$. Thus,

$$\Pi(3,\frac{1}{2}) = \theta^3 + 2\theta^2(1-\theta)(1-x) + 2\theta(1-\theta)^2(1-x)^2 = \frac{97}{200} < \frac{150}{200}$$

As with an increase in θ, examples where Π decreases in n due to the h type playing a riskier strategy can easily be constructed.

■

Proof of Proposition 4: From Dekel and Scotchmer (1999), Proposition 3, we know that there exists a finite n, denoted n^*, such that for all n larger than n^*, $(risky, risky)$ is the unique equilibrium. It follows that $(risky, risky)$ is the only equilibrium for an infinite number of contestants. Consequently, with an infinite number of contestants, the winner has output equal to z_4, with probability 1. By the law of large numbers, the share of H agents that achieve z_4 is just y, and the share of L agents that achieve z_4 is equal to x. Thus $\Pi(\infty) = \dfrac{\theta y}{\theta y + (1-\theta)x}$. Now consider $\theta = \frac{1}{2}, x = \frac{1}{5}, y = \frac{1}{4}$. With those parameter values, we have $\Pi(\infty) = \frac{5}{9} < \frac{3}{4} = \Pi(2)$. ■

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